International Journal of Theoretical Physics, Vol. 4, No. 6 (1971), pp. 403–425

Photons, Neutrinos, Electrons and Baryons in a Unified Spinor Field Theory of Relativity

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Received: 17 *March*, 1971

1. *Introduction*

Einstein explained gravitation by assuming that space-time was curved and that masses followed geodesics in the curved space-time. In particular he proposed that the contracted Ricci tensor of the curvator tensor was determined by the energy–momentum–tension tensor. This provided a good model for a universe of neutral particles and gave a very useful explanation of gravitation.

The theory failed to explain the physical world in three respects.

(1) It did not explain electromagnetic forces.
(2) It left no room for quantum effects.
(3) It gave no explanation as to the nature of matter.

Kaluza provided an answer to the first deficiency. He proposed that the four-dimensional space-time should be extended by a fifth cylindrical dimension of constant (small) period. The connectivity of this fifth dimension introduced an extra form of space curvature which could be represented by a 6-vector in the homomorphic 4-space, and this 6-vector could be identified with the electromagnetic field. Uncharged particles followed geodesics independent of this fifth dimension, while charged particles followed geodesics which continually rotated round this fifth dimension.

This explained electromagnetism by space curvature in the same way as Einstein explained gravitation. The general judgement at the time was reserved, in that there was some doubt as to whether the results obtained were sufficient to outweigh the complication of a fifth dimension. Pauli (1958) wrote:

> 'The question of whether Kaluza's formalism has any future in physics is thus leading to the more general unsolved main problem of accomplishing a synthesis between the general theory of relativity and quantum mechanics.'

Passing over the second deficiency for the time being, consider the third, namely the failure to explain the nature of matter. There is ample evidence to show that matter has the nature of a wave. Hence, if it is hoped eventually to give a complete explanation of the physical world there must be in this explanation a wave, and hence a wave function. This wave function must be transformed by a change of coordinates, and hence this transformation must give a matrix representation of the orthogonal group. Hence the wave function must be either a scalar, a tensor or a spinor. If it were a scalar or a tensor, then all concomitants would be scalars or tensors and there would be no room in mathematical physics for spinors. The simplest possible hypothesis, therefore, is that the wave function is a basic spinor.

In 4-space with signature 2, the basic spinor can be real. In 5-space of signature 3 the basic spinor is necessarily complex. It seems necessary in later calculation to assume that the basic spinor is complex, and thus the consistency with Kaluza space-time is reassuring. The considered concomitants of the basic spinor will be the Hermitian concomitants.

The simplest possible wave equation that can be considered is the first-order invariant linear differential equation

$$\operatorname{curl} W = 0$$

using a nomenclature which will be defined in Sections 3 and 4.

The basic spinor has two Hermitian concomitants which form a 4-vector and a 6-vector.

The time component of the 4-vector is the norm of the spinor $\bar{\tilde{W}}W$. It would be very reasonable to associate this with matter density. This would imply that the energy momentum tensor was $Q_i Q_j$.

The 6-vector concomitant F_{ij} might very reasonably be associated with the electromagnetic field. Hence we are led, almost inevitably, to the following hypotheses.

Hypotheses

(1) There exists a distribution of complex basic spinors W in a Kaluza 5-space.

(2) The spinors satisfying a basic linear differential equation

$$\operatorname{curl} W = 0$$

(3) The space curvatures are determined by the 6-vector concomitant $F_{ij} = \bar{\tilde{W}}TX_i X_j W$, and by the 4-vector concomitant $Q_i = \bar{\tilde{W}}TX_i W$.

There is also a skew Hermitian concomitant

$$P_i = \bar{\tilde{W}}T\phi X_i W,$$

and the third hypothesis might be modified to depend also upon this. This conforms more closely with Einstein's General Theory, but the physical evidence is not conclusive.

Except for this last ambiguity, these hypotheses lead to a precise mathematical model. The mathematical properties of this model are extremely complex and only first approximations have been calculated. However, these first approximations seem to show a remarkably close correspondence with the physical world. Quantised light radiation, neutrinos, a wave packet electron with mechanical and magnetic spins, the Pauli exclusion principle, frequency of spectral lines and a model for baryons, are all features of this mathematical model.

On grounds of probability alone, it would seem to be virtually impossible that a model which did not have some real physical significance could lead to such a close fit with the physical world.

2. *The Kaluza Model*

A brief description is first given of the Kaluza model (1921). It has been abstracted from Pauli's (1958) account to which the reader is referred for further details.

One considers a 5-space with a cylindrical metric

$$ds^2 = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu \qquad (\mu, \nu, = (1, 2, 3, 0, 5))$$

The fifth dimension has a small period in which one takes $\gamma_{55} = 1$. If the metric is put in the form

$$ds^2 = (dx^5 + \gamma_{i5}\, dx^i)^2 + g_{i\kappa}\, dx^i\, dx^\kappa \qquad (i, \kappa = 1, 2, 3, 0)$$

then transformations of the form

$$x'^5 = x^5 + f(x_1, x_2, x_3, x_0)$$

leave

$$\frac{\partial \gamma_{k5}}{\partial x^i} - \frac{\partial \gamma_{i5}}{\partial x^k} = f_{ik} \tag{2.1}$$

invariant. This represents a form of space curvature, and is identified with the electromagnetic field.

The homomorphic 4-space can be identified with the Riemannian space of Einstein's General Relativity, and thus an uncharged particle follows a geodesic which does not involve the fifth dimension.

For a geodesic inclined at an angle θ towards the fifth dimension to an uncharged particle geodesic, the path is influenced by the terms γ_{i5} in the metric, and calculations show that the deflection of the projection of the geodesic onto the homorphic 4-space is precisely the same as the deflection from a geodesic path which would be caused by an electromagnetic field

$$f_{ik} = \frac{\partial \gamma_{k5}}{\partial x^i} - \frac{\partial \gamma_{i5}}{\partial x^k}$$

on a particle with charge-mass ratio $e/m = \tan\theta$.

3. *The Spinor Frame*

The definition of spinors depends on a frame of anticommuting matrices. These can be real for four dimensions of signature 2, which case will be considered first.

For the metric

$$ds^2 = (dx_1)^2 + (dx_2)^3 + (dx_3)^2 - (dx_0)^2$$

the following set is chosen satisfying

$$X_1{}^2 = X_2{}^2 = X_3{}^2 = 1 = -X_0{}^2$$

$$X_1 = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} & -1 & & \\ -1 & & & \\ & & & 1 \\ & & 1 & \end{bmatrix},$$

$$X_3 = \begin{bmatrix} & & & -1 \\ & & -1 & \\ & -1 & & \\ -1 & & & \end{bmatrix}, \quad X_0 = \begin{bmatrix} & 1 & & \\ -1 & & & \\ & & & 1 \\ & & -1 & \end{bmatrix} \tag{3.1}$$

with $X^i = X_i$ $(i = 1, 2, 3)$, $X^0 = -X_0$.

Then

$$(x_1 X_1 + x_2 X_2 + x_3 X_3 + x_0 X_0)^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 - (x_0)^2 \tag{3.2}$$

For a Lorentz transformation

$$x_i' = \xi_i^j x_j$$

let

$$X_i' = \xi_i^j X_j$$

Then there exists a matrix U, unique apart from sign such that

$$X_i' = U^{-1} X_i U, \qquad UX_i' = X_i U \tag{3.3}$$

The matrix $T = X_0$ clearly transforms each X_i into $-X^i = -\tilde{X}_i$,

$$TX_i = -\tilde{X}_i T \tag{3.4}$$

the symbol $\sim$ denoting transposition.

A 4-rowed vector W is called a *basic spinor* if, corresponding to the above Lorentz transformation it transforms so that

$$W' = UW \tag{3.5}$$

Clearly

$$\tilde{W} = \tilde{W}\tilde{U}$$

The relation between $\tilde{U}$ and U can be obtained as follows:

$$\begin{aligned} X_i'(T\tilde{U}TU) &= -T\tilde{X}_i'\tilde{U}TU = -T\tilde{U}\tilde{X}_i TU \\ &= T\tilde{U}TX_i U = (T\tilde{U}TU)X_i' \end{aligned}$$

Thus $T\tilde{U}TU$ commutes with each X_i' and thus with every matrix. Hence it is a scalar multiple of the unit matrix. The arbitrary scalar factor in U can be adjusted so that

$$T\tilde{U}TU = -I, \qquad \tilde{U}TU = T \tag{3.6}$$

at least for transformations which do not reverse the direction of time.

4. *Concomitants of Degree* 2

The types of concomitants which are bilinear in a complex spinor W and its complex conjugate $\bar{W}$ can be determined by characteristic analysis, the formula being

$$\Delta\Delta = \{0\} + \{1\} + \{1^2\} + \{1^3\} + \{1^4\} \tag{4.1}$$

For quadratic terms in a real spinor the formula is

$$\Delta \otimes \{2\} = \{1\} + \{1^2\} \tag{4.2}$$

Ignoring the difference between an absolute invariant corresponding to $\{0\}$ and a relative invariant corresponding to $\{1^4\}$, and similarly not discriminating between $\{1\}$ and $\{1^3\}$, the concomitant consist of two invariants, two 4-vectors and a 6-vector.

$$\text{Putting } \phi = X_1 X_2 X_3 X_0 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \tag{4.3}$$

which behaves as a relative invariant, the concomitants are

$$\begin{aligned} I &= \tilde{\bar{W}}TW, \qquad & J &= \tilde{\bar{W}}T\phi W \\ Q_i &= \tilde{\bar{W}}TX_i W, \qquad & P_i &= \tilde{\bar{W}}T\phi X_i W \end{aligned} \tag{4.4}$$

$$F_{ij} = \tilde{\bar{W}}TX_i X_j W$$

To show that e.g., Q_i transforms as a 4-vector, note that

$$\begin{aligned} Q_i' &= \tilde{\bar{W}}'TX_i W' = \tilde{\bar{W}}\tilde{U}TX_i UW \\ &= \tilde{\bar{W}}(\tilde{U}TU)(U^{-1}X_i U)W = \tilde{\bar{W}}TX_i' W \end{aligned}$$

The other concomitants transform similarly.

If $\bar{\tilde{W}} = [\alpha, \beta, \gamma, \delta]$ the concomitants in full are

$$
\begin{aligned}
I &= \bar{\alpha}\beta - \bar{\beta}\alpha + \bar{\gamma}\delta - \bar{\delta}\gamma, & J &= \bar{\alpha}\gamma - \bar{\gamma}\alpha - \bar{\beta}\delta + \bar{\delta}\beta \\
P_i &= \bar{\alpha}\gamma - \bar{\gamma}\alpha + \bar{\beta}\delta - \bar{\delta}\beta, & P_2 &= \bar{\alpha}\delta - \bar{\delta}\alpha - \bar{\beta}\gamma + \bar{\gamma}\beta \\
P_3 &= -\bar{\alpha}\beta + \bar{\beta}\alpha + \bar{\gamma}\delta - \bar{\delta}\gamma, & P_0 &= \bar{\alpha}\delta - \bar{\delta}\alpha + \bar{\beta}\gamma - \bar{\gamma}\beta \\
Q_1 &= \bar{\alpha}\beta + \bar{\beta}\alpha + \bar{\gamma}\delta + \bar{\delta}\gamma, & Q_2 &= \bar{\alpha}\alpha - \bar{\beta}\beta - \bar{\gamma}\gamma + \bar{\delta}\delta \\
Q_3 &= \bar{\alpha}\gamma + \bar{\gamma}\alpha - \bar{\beta}\delta - \bar{\delta}\beta, & Q^0 &= \bar{\alpha}\alpha + \bar{\beta}\beta + \bar{\gamma}\gamma + \bar{\delta}\delta \\
F_{01} &= -\bar{\alpha}\alpha + \bar{\beta}\beta - \bar{\gamma}\gamma + \bar{\delta}\delta, & F_{02} &= \bar{\alpha}\beta + \bar{\beta}\alpha - \bar{\gamma}\delta - \bar{\delta}\gamma \\
F_{03} &= \bar{\alpha}\delta + \bar{\delta}\alpha + \bar{\beta}\gamma + \bar{\gamma}\beta, & F_{23} &= \bar{\alpha}\delta + \bar{\delta}\alpha - \bar{\beta}\gamma - \bar{\gamma}\beta \\
F_{31} &= -\bar{\alpha}\gamma - \bar{\gamma}\alpha - \bar{\beta}\delta - \bar{\delta}\beta, & F_{12} &= \bar{\alpha}\alpha + \bar{\beta}\beta - \bar{\gamma}\gamma - \bar{\delta}\delta
\end{aligned}
\tag{4.5}
$$

The 6-vector F_{ij} being associated with the electromagnetic field, it is convenient to write

$$F_{0i} = E_i, \qquad F_{23} = H_1, \qquad F_{31} = H_2, \qquad F_{12} = H_3 \tag{4.6}$$

The alternating tensor Γ^{ijkp} being defined as $+1$ if (i,j,k,p) is a positive permutation of $(1,2,3,0)$ or -1 if a negative permutation, but zero if two suffixes are equal, may be used to convert R_{ij} into its dual form

$$\hat{F}^{ij} = \Gamma^{ijkp} F_{kp} \tag{4.7}$$

This has the effect of interchanging the electric and magnetic terms with a change of sign in one case

$$\hat{E}_i = H_i, \qquad \hat{H}_i = -E_i \tag{4.8}$$

The concomitants Q_i and F_{ij} are real, and these correspond to the concomitants of a real spinor. The other concomitants I, J and P_i are pure imaginary.

The Operators $\exp(t\phi)$, $\exp(it\phi)$

Since $\phi^2 = -1$,

$$\exp(t\phi) = \cos t + \phi \sin t$$

The transformation $W' = \exp(t\phi)\, W$ changes

$$
\begin{aligned}
I \text{ into } I' &= I \cos 2t - J \sin 2t \\
J \text{ into } J' &= J \cos 2t + I \sin 2t \\
E_i \text{ into } E_i' &= E_i \cos 2t - H_i \sin 2t \\
H_i \text{ into } H_i' &= H_i \cos 2t + E_i \sin 2t
\end{aligned}
\tag{4.9}
$$

but leaves P_i and Q_i unaltered.

The operator

$$\exp(it\phi) = \cosh t + i\phi \sinh t$$

has a different effect because of the extra change of sign in taking the transposed conjugate. The concomitant I, J, F_{ij} are unaltered, but

$$W' = \exp(it\phi)\,W$$

changes

$$\begin{aligned} &P \text{ into } P\cosh t - iQ\sinh t \\ &Q \text{ into } Q\cosh t - iP\sinh t \end{aligned} \tag{4.10}$$

5. *Syzygies*

The set of concomitants of a set of ground forms are not algebraically independent, but the forms are connected with each other by a set of relations which are called syzygies.

For concomitants of a real basic spinor these syzygies are easily seen to be

$$\begin{aligned} Q_1^2 + Q_2^2 + Q_3^2 &= Q_0^2 \\ E_1^2 + E_2^2 + E_3^3 &= Q_0^2 \\ H_1^2 + H_2^2 + H_3^2 &= Q_0^2 \\ Q_1 E_1 + Q_2 E_2 + Q_3 E_3 &= 0 \\ Q_1 H_1 + Q_2 E_2 + Q_3 E_3 &= 0 \\ E_1 H_1 + E_2 H_2 + E_3 H_3 &= 0 \end{aligned} \tag{5.1}$$

Thus, relative to any given time axis a single real basic spinor defines a set of three mutually orthogonal 3-vectors of equal magnitude. The physical significance of a real basic spinor may therefore be interpreted as an orientation of 3-space relative to any given time axis, together with a scalar Q_0 which transforms as the time component of a 4-vector.

For a complex spinor the syzygies are much more complicated. To make sure of a complete set characteristic analysis is used. The Hermitian concomitants correspond to $\Delta.\Delta$. The powers and products of degree 2 of these, to (Littlewood, 1958)

$$(\Delta.\Delta)\otimes\{2\} = (\Delta\otimes\{2\})(\Delta\otimes\{2\}) + (\Delta\otimes\{1^2\})(\Delta\otimes\{1^2\})$$

The terms corresponding to $(\Delta\otimes\{2\})(\Delta\otimes\{2\})$ are concomitants of degree 2 in the spinor and of degree 2 in the conjugate. The remaining terms corresponding to

$$\begin{aligned} (\Delta\otimes\{1^2\})(\Delta\otimes\{1^2\}) &= (2[0]+[1])(2[0]+[1]) \\ &= 5[0]+4[1]+[2]+[1^2] \end{aligned}$$

must therefore be zero, and represent the syzygies, of which there are five invariants, four of type [1], and two others of types [2] and $[1^2]$ representatively.

These have been obtained elsewhere (Littlewood, 1969, p. 120), and will merely be quoted here;

$$\begin{aligned} &P_i Q^i = 0, && -F_{ij}F^{ij} \equiv E^2 - H^2 = I^2 - J^2 \\ &\tfrac{1}{2}\hat{F}_{ij}F^{ij} \equiv E.H = -IJ, && P_i P^i = Q_i Q^i = I^2 + J^2 \end{aligned} \tag{5.2}$$

$$\begin{aligned} &P^i F_{ij} = JQ_j, && P^i \hat{F}_{ij} = IQ_j \\ &Q^i F_{ij} = -JP_j, && Q^i \hat{F}_{ij} = -IP_j \end{aligned} \tag{5.3}$$

$$P_i Q_j - P_j Q_i = I\hat{F}_{ji} + JF_{ji} \tag{5.4}$$

$$\tfrac{1}{2}(F_{ij}F_k{}^i + \hat{F}_{ij}\hat{F}_k{}^i) = P_j P_k + Q_j Q_k - \tfrac{1}{4}g_{jk}(P^i P_i + Q^i Q_i) \tag{5.5}$$

6. *Relation Between Spinors and* 6-*Vectors*

Theorem 1

Given any 6-vector R_{ij} there exists a basic spinor W such that $F_{ij} = \tilde{W}TX_i X_j W$.

The matrix $[F_{st}]$ has determinant

$$|F_{st}| = \Delta^2$$

where

$$\Delta = F_{12}F_{30} + F_{23}F_{10} + F_{31}F_{20}$$

Replacement of F_{st} by its dual $\hat{F}_{st}$ changes Δ into $-\Delta$. Thus the function Δ of $[F_{st} + \lambda\hat{F}_{st}]$ changes sign between $-\infty$ and 0 and between 0 and $+\infty$. Thus

$$|F_{st} + \lambda\hat{F}_{st}| = 0$$

has one negative and one positive root. Choosing a Lorentz frame so that the factors of zero of the matrix for these two roots correspond to the $x_1 x_2$-plane and the $x_3 x_0$-plane respectively, this implies that the only non-zero terms of R_{ij} are

$$E_3 = k_1 \qquad H_3 = k_2$$

Consider a spinor for which α and δ are real, and

$$\beta = i\alpha, \qquad \gamma = i\delta$$

then the 6-vector concomitant has the following non-zero terms

$$E_3 = -4a\delta, \qquad H_3 = 2(\alpha^2 - \delta^2)$$

A suitable choice of α and δ satisfies the theorem.

Exception occurs only if $|F_{st} + \lambda\hat{F}_{st}|$ is identically zero. In this case E and H are of equal magnitude and orthogonal. The 6-vector is then the concomitant of a real spinor.

The complex spinor has eight independent terms, the 6-vector has only six. There is an ambiguity involving two terms to be accounted for. The first ambiguity is very simple.

Theorem 2

All concomitants of a complex spinor are independent of a change of phase $W' = \exp(i\theta)\,W$.

It is only necessary to point out that $\bar{W}' = \exp(-i\theta)\,W$ and in the concomitants the factors $\exp(i\theta)$ and $\exp(-i\theta)$ cancel.

The other ambiguity is more interesting. If W is replaced by $W' = \exp(it\phi)\,W_0$ as explained in Section 4, then the concomitants I, J, and the 6-vector R_{ij} are unchanged. Hence for a specified F_{ij} there is an arbitrary factor $\exp(it\phi)$, which constitutes the other ambiguity.

There is, however, a change in the 4-vectors P_i and Q_i, namely (4.10)

$$P' = P\cosh t - iQ\sinh t$$
$$Q' = Q\cosh t - iP\sinh t$$

Notice that if R_{ij} is given there is not one, but a continuous infinity of time axes which make E and H collinear, i.e. relative to which there is no flow of momentum. These will be called *apparent rest frames*.

The choice of the factor $\exp(it\phi)$ picks out exactly one of these frames relative to which Q_i is in the direction of the time axis and P_i is purely spacial.

The energy–momentum–tension tensor may be expressed as

$$\begin{aligned} T_{jk} &= \tfrac{1}{2}(F_{ij}F_k^{\,o} + \hat{F}_{ij}\hat{F}_k^{\,i}) \\ &= (P_jP_k - \tfrac{1}{4}g_{jk}P_iP^i) + (Q_jQ_k - \tfrac{1}{4}g_{jk}Q_iQ^i) \end{aligned}$$

by using the syzygy (5.5).

This effects a separation of T_{jk} into two parts, one corresponding to the stress, the other to the energy, with a corresponding separation of the momentum terms.

This separation is not uniquely determined by F_{ij}, but the choice of one of the apparent rest frames to form the rest frame fixes the arbitrary factor $\exp(it\phi)$ and renders the separation unique.

This separation of T_{jk} does open a new possibility of variation in Einstein's general theory. To Einstein T_{jk} seemed indivisible, which left him no option but to make the Ricci tensor dependent upon it. If it is separable, however, it is possible to make the Ricci tensor depend on $k_1P_iP_j + k_2Q_iQ_j$ and determine whether or not the best fit with the physical world is obtained by putting $k_1 = k_2$. The term Q_iQ_j alone would explain gravitation. Perhaps the instability of a very large gravitational mass which is one of the paradoxes of Einstein's General Theory could be explained away in this manner.

6. *The Differential Equation*

Differentiation in space-time involves the operator $\partial/\partial x_i$ which transforms like a vector of type $\{1\}$. The derivatives of a basic spinor correspond to

$$\Delta[1] = [\tfrac{3}{2}, \tfrac{1}{2}] + [\tfrac{1}{2}, \tfrac{1}{2}]$$

Part of the derivative is therefore a basic spinor, which is easily seen to be

$$\text{curl } W = \frac{\partial}{\partial x_i} X_i W$$

In three, four or five dimensions the operator will be denoted by

$$\nabla = \frac{\partial}{\partial x_i} X_i, \qquad (i = 1, 2, 3)$$

$$\text{curl}_4 = \frac{\partial}{\partial x_i} X_i, \qquad (i = 1, 2, 3, 0)$$

$$\text{curl}_5 = \frac{\partial}{\partial x_i} X_i, \qquad (i = 1, 2, 3, 0, 5)$$

The extension to five dimensions requires the additional anticommuting matrix X_5, uniquely determined apart from sign as

$$X_5 = i\phi = iX_1 X_2 X_3 X_0 = \begin{bmatrix} & & & i \\ & & i & \\ & -i & & \\ -i & & & \end{bmatrix}$$

Then if $\tilde{W} = [\alpha, \beta, \gamma, \delta]$ $\text{curl}_5 \tilde{W} = [\alpha', \beta', \gamma', \delta']$, $\text{curl}_5 W$ is given in detail by

$$\begin{aligned} \alpha' &= \alpha_1 - \beta_2 - \delta_3 + \beta_i + i\delta_5 \\ \beta' &= -\beta_1 - \alpha_2 - \gamma_3 - \alpha_0 + i\gamma_5 \\ \gamma' &= \gamma_1 + \delta_2 - \beta_3 + \delta_0 - i\beta_5 \\ \delta' &= -\delta_1 + \gamma_2 - \alpha_3 - \gamma_0 - i\alpha_5 \end{aligned}$$

where α_i denotes $\partial\alpha/\partial x_i$.

7. *Light Waves and Neutrino Radiation*

Light waves and neutrino radiation correspond to solutions of curl $W = 0$, which do not involve the fifth dimension, so that $\partial W/\partial x_5 = 0$. Hence $\text{curl}_4 W = 0$. In the first investigation space curvature will be ignored.

The equation $\text{curl}_4 W = 0$ is then clearly a wave equation. Seeking solutions as plane waves propagated in the x_2-direction so that $\partial W/\partial x_1 = \partial W/\partial x_3 = 0$, thus gives

$$\begin{aligned} \beta_2 &= \beta_0, & \gamma_2 &= \gamma_0 \\ \alpha_2 &= -\alpha_0, & \delta_2 &= -\delta_0 \end{aligned}$$

It is convenient to use c to denote the theoretical velocity of light. Units are used so that $c = 1$, but it is sometimes more descriptive physically, to use c instead of unity. The solutions of $\text{curl}_4 W = 0$ correspond to waves propagated with velocity c.

Solutions can be expressed in terms of monochromatic waves such as

$$\alpha = k\cos\nu(x_2 - x_0), \qquad \delta = k\sin\nu(x_2 - x_0), \qquad \beta = \gamma = 0 \tag{7.1}$$

This gives $Q_2 = Q_0 = k^2$,

$$\begin{aligned} E_1 &= -k^2\cos 2\nu(x_2 - x_0), & E_3 &= k^2\sin 2\nu(x_2 - x_0) \\ H_1 &= k^2\sin 2\nu(x_2 - x_0), & H_3 &= k^2\cos 2\nu(x_2 - x_0) \end{aligned} \tag{7.2}$$

as the only non-zero terms of the concomitants.

This describes exactly the energy momentum and electromagnetic field associated with circularly polarised light.

Using complex spinors it is possible to obtain a similar solution of $\mathrm{curl}_4\, W = 0$ which represents the same energy flow, but with a zero electromagnetic field. Thus

$$\alpha = -i\delta = k\exp[i\nu(x_2 - x_0)] \tag{7.3}$$

It would seem fitting to interpret this as neutrino radiation.

Weyl's (1929) equations for the neutrino in this nomenclature are given by

$$\mathrm{curl}\, W = 0, \qquad W = X_5 W$$

It will now be shown that, with an ambiguity of sign, the second equation is exactly the condition that the 6-vector concomitant F_{ij} is zero. This provides some corroboration for the hypothesis that F_{ij} represents the electromagnetic field.

Theorem

A necessary and sufficient condition that the concomitant R_{ij} of a complex basic spinor W should be zero is that $W = \pm X_5 W$.

Firstly to prove that the condition is sufficient, assume

$$W = X_5 W, \qquad \tilde{\bar{W}} = \tilde{\bar{W}}\tilde{\bar{X}}_5 = \tilde{\bar{W}} X_5$$

Then

$$\begin{aligned} F_{ij} &= \tilde{\bar{W}} T X_i X_j W = \tilde{\bar{W}} X_5 T X_i X_j X_5 W \\ &= -\tilde{\bar{W}} T X_i X_j X_5 X_5 W = -W T X_i X_j W \\ &= -F_{ij} = 0 \end{aligned}$$

The case $W = -X_5 W$ is essentially similar.

To prove necessity put $W = W_1 + iW_2$, where W_1 and W_2 are real spinors. A rotation of axes will bring W_1 to the form

$$\tilde{\bar{W}}_1 = [k, 0, 0, 0]$$

and let

$$\tilde{\bar{W}}_2 = [\alpha, \beta, \gamma, \delta]$$

The concomitants Q_i and R_{ij} of W are equal to the sum of the same concomitant of W_1 and W_2 respectively.

Hence if $F_{ij} = 0$

$$F_{03} - F_{23} = 4\beta\gamma = 0, \quad F_{01} + F_{12} = 2\beta^2 - 2\gamma^2 = 0$$

so that $\beta = \gamma = 0$.

Also

$$F_{03} + F_{23} = 4\alpha\delta = 0, \qquad F_{01} = -k^2 - \alpha^2 + \delta^2 = 0$$

so that $\alpha = 0$, $\delta = \pm k$, and

$$W_2 = \pm iX_5 W_1, \qquad W = \pm iX_5 W$$

A field corresponding to $W = X_5 W$ is taken to represent a neutrino, and for $W = -X_5 W$ an antineutrino.

A typical expression for neutrino radiation would be

$$\tilde{\overline{W}} = \{k \exp [i\nu(x_2 - x_0)], 0, 0, - ik \exp [i\nu(x_2 - x_0)]\} \tag{7.4}$$

and for antineutrino radiation

$$\tilde{\overline{W}} = \{k \exp [-i\nu(x_2 - x_0)], 0, 0, ik \exp [-i\nu(x_2 - x_0)\} \tag{7.5}$$

Notice that if these two waves are superimposed, the result will be exactly that previously obtained (7.1) for circularly polarised light.

Moret-Bouilly (1966) has pointed out that a circularly polarised photon could be regarded as the fusion of a neutrino and an antineutrino.

8. *Quantisation of Radiation*

These infinite plane waves can only exist if space curvature is neglected. Next will be considered the effect of gravitational curvature. This can have the effect of condensing the infinite plane waves into finite wave packets.

Examination of the Schwartzchild exact solution for a single massive particle of Einstein's equation $G_{\mu\nu} = 0$, indicates that this solution may be obtained by modifying the metric of a flat space-time so that the time rate is slowed down according to the gravitational potential at a point, added to a modification of the special metric in the direction of the gravitational field. The main gravitational effect is a consequence of the slowing down of the time rate. As an approximation to the analysis of the gravitational effect, it will just be assumed that the time rate is slowed down by a factor equal to the gravitational potential ($\leqslant 1$). This effect has been experimentally verified (Mössbauer, 1958), and thus the conclusion is independent of the truth of Einstein's theory.

Using x_1, x_2, x_3, t for the coordinates in flat space-time the local time coordinate x_0 satisfies

$$dx_0 = \phi\, dt \qquad (\phi \leqslant 1), \qquad dt = \psi\, dx_0 \tag{8.1}$$

with ϕ the gravitational potential and $\psi = \phi^{-1} \geqslant 1$.

If a wave packet is to remain coherent without dissipating itself by self interference, then the time frequency must be constant with respect to the universal time t, but will vary locally according to the gravitational potential.

For a wave propagation in the x_2-direction put

$$W = W' + W''$$

where

$$W' = \tfrac{1}{2}(1 - X_2 X_0)\, W, \qquad W'' = \tfrac{1}{2}(1 + X_2 X_0)\, W \tag{8.2}$$

so that the momentum of W' corresponds to a velocity c in the x_2-direction, and of W'' to velocity c in the reverse direction.

It will be shown that the effect of the varying gravitational potential is to cause a partial reflection of the incident ray W' into the reflected ray W'', and conversely.

Provided that $W'' \neq 0$, the momentum of the wave packet include terms from W'' in the reverse direction, and will thus correspond to a velocity $< c$.

This implies that the group velocity of the wave packet is $< c$. It is difficult to approximate to the gravitational effect of a moving body. But since it is a basic principle of relativity that there is a valid frame of reference relative to which any given body is at rest, always assuming that it is not already travelling with velocity c, therefore we can make a Lorentz transformation to a frame of reference relative to which the wave packet is at rest.

For this frame of reference W' consists of a narrow pencil of light propagated in the x_2-direction and being continually reflected back along its path in the $-x_2$-direction until the reflected pencil W'' has equal intensity to the incident-pencil. The effect of the gravitational field of this wave packet on itself is examined.

Operating on the equation

$$\left(X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3} + X_0 \frac{\partial}{\partial x_0}\right) W = 0$$

with the operators $\tfrac{1}{2}(1 - X_2 X_0)$ and $\tfrac{1}{2}(1 + X_2 X_0)$ gives

$$\begin{aligned} \left(X_1 \frac{\partial}{\partial x_1} + X_3 \frac{\partial}{\partial x_3}\right) W' &= -\left(X_2 \frac{\partial}{\partial x_2} + X_0 \frac{\partial}{\partial x_0}\right) W'' \\ \left(X_1 \frac{\partial}{\partial x_1} + X_3 \frac{\partial}{\partial x_3}\right) W'' &= -\left(X_2 \frac{\partial}{\partial x_2} + X_0 \frac{\partial}{\partial x_0}\right) W' \end{aligned} \tag{8.3}$$

Assume a periodicity factor $\exp(i\nu t)$, with a factor $\exp(ik\nu x_2)$ in the x_2-direction. Then, since $dt = \psi\, dx_0$, $\psi^{-1} = \phi =$ self gravitational potential,

$$\left(X_1 \frac{\partial}{\partial x_1} + X_3 \frac{\partial}{\partial x_3}\right) W'' = -i\nu X_2 (k - \psi)\, W' \tag{8.4}$$

since $X_2 W' = -X_0 W'$.

In the case of circularly polarised light radiation the factor i in the index could be replaced by $X_2 X_0$, the calculation being otherwise unaltered.

Assume that the wave packet is substantially confined to the neighbourhood of the line $x_1 = x_3 = 0$, and that in the plane $x_2 = 0$, W' is equal to a scalar multiple of the value W would take for an infinite plane wave in flat space time. That is to say, in the plane $x_2 = x_0 = 0$, $W' = F\Omega$, where Ω is a fixed spinor, necessarily of the form

$$\tilde{\Omega} = [k', 0, 0, k'']$$

while F is a scalar function of position. It would be simpler, and essentially reasonable to assume that F has circular symmetry, so that

$$W' = f(r)\Omega, \qquad r^2 = (x_1)^2 + (x_3)^2 \tag{8.5}$$

Using Green's theorem for a large circle Γ centre origin in the plane $x_2 = x_0 = 0$, enclosing a disc D,

$$\int_\Gamma X_1 W'' dx_3 - X_3 W'' dx_1 = 0 = \iint_D \left(X_1 \frac{\partial}{\partial x_1} + X_3 \frac{\partial}{\partial x_3}\right) W'' dx_1 dx_3$$

$$= -i\nu X_2 \iint_D (k - \psi) W' dx_1 dx_3 \tag{8.6}$$

Thus

$$k = \frac{\iint_D \psi W' dx_1 dx_3}{\iint_D W' dx_1 dx_3}$$

and $k = \psi_0$, the mean value of ψ in the plane weighted by the magnitude of W'.

This gives

$$\left(\frac{\partial^2}{\partial x_1{}^2} + \frac{\partial^2}{\partial x_3{}^2}\right) W' = \left(\frac{\partial^2}{\partial x_0{}^2} - \frac{\partial^2}{\partial x_2{}^2}\right) W' = (\psi_0{}^2 - \psi^2)\nu^2 W' \tag{8.7}$$

Substituting $W' = f(r)\Omega$

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} = (\psi_0{}^2 - \psi^2)\nu^2 f \tag{8.8}$$

Since $\psi \to 1$ as $r \to \infty$, clearly $f \to 0$ as $r \to \infty$ in the manner of $\exp[-(\psi_0 - 1)\nu r]$. It will be assumed that the only maximum of $f(\nu)$ in the plane is at $r = 0$. This implies that if $\psi = \psi_0$ at $r = r_0$, then $\psi < \psi_0$ for $r > r_0$, and that $\psi > \psi_0$ for $r < t_0$. Then

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{dr} > 0 \qquad \text{for } r > r_0 \tag{8.9}$$

and putting $v = \log r$

$$\frac{\partial^2 f}{\partial u^2} > 0 \qquad \text{for } r > r_0 \tag{8.10}$$

Thus $f(r)$ behaves like a negative exponential type of curve for $r > r_0$.

This implies that if the potential deficiency is confined to the neighbourhood of the x_2-axis, then the wave W' will be substantially confined to the neighbourhood of the x_2-axis, and the wave packet would be confined to a tight pencil.

The assumption that W involves a factor $\exp(ik\nu x_2)$ implies that this pencil would be of infinite length. This in turn would lead to an infinite potential. Hence this assumption must be modified by assuming a superposition of such waves with slightly varying x_2-periods, so that the phases would interfere at a distance from $x_2 = 0$, thus producing a wave packet of finite length.

Stability

It is clear that the gravitational track acts as a wave guide in that wherever ψ becomes less than ψ_0 the intensity $f(r)$ of W' falls off exponentially as r increases. Suppose that there was present a wave whose propagation direction made a small angle ϵ with the direction of the wave guide. For temporal stability the time frequency must remain invariant. The angle ϵ would cause the x_2-period to be increased by a factor $\sec\epsilon$, replacing ψ_0 by $\psi_0 \cos\epsilon$. Then equations (8.8) and (8.10) would be replaced by

$$\frac{\partial^2 f}{\partial u^2} = \exp(2u)(\psi_0{}^2\cos^2\epsilon - \psi^2)\nu^2 f$$

Hence, so long as $\cos\epsilon > 1/\psi_0$, this would make $\partial^2 f/\partial u^2 > 0$ for $r > r_0'$, and the magnitude of the wave would still tend to zero exponentially as r increased to ∞.

Thus containment by the gravitational wave guide would still be effective so long as the direction of propagation made an angle $< \sec^{-1}\psi_0$ with the direction of the gravitational wave guide.

This would ensure stability for the wave pocket. It would also allow the simple wave with fixed x_2-periodicity to be replaced by a pencil of waves with directions of propagation slightly deviating from the x_2-direction, so as to form a wave packet of finite length.

The complete mathematical analysis of such a stationary wave packet is extremely complicated, although it seems quite clear that such a solution exists. It could be that the stability, with equality of magnitude of the incident and reflected waves, is only obtainable for a fixed energy and fixed frequency. If this were the case, since every light quantum would be obtainable from this solution by a Lorentz transformation, and since energy and frequency transform in the same manner, it would follow that the energy for a stable photon is exactly proportional to its frequency.

As well as a stable photon there must also exist radiation which is not quantised for stability. This would explain some of the paradoxes which led people to think that a deterministic solution of the physical world was impossible.

Neutrino radiation would be similarly quantised.

9. *The Electron*

As has been seen, the equation $\operatorname{curl} W = 0$, with $\partial W/\partial x_5 = 0$ leads generally to waves propagated with velocity c. It is evident, however, from modern physics, that particles should also be regarded as wave packets, in particular the electron. It would be natural to suppose that for such a wave packet $\partial W/\partial x_5 \neq 0$. This would also follow from the fact that an electron is deflected by an electromagnetic field.

Take the unit of length so that the fifth-dimensional period is 2π in the x_5-direction. The simplest case is that W depends on x_5 by the introduction of a factor $\exp(ix_5)$, or $\exp(-ix_5)$.

Ignoring space curvature there are static solutions of $\operatorname{curl}_5 W = 0$ of the form

$$\begin{aligned}\tilde{W} = \{&K\exp[i(x_0+x_5)],\ K'\exp[i(x_0+x_5)],\ -iK\exp[i(x_0+x_5)],\\ &iK'\exp[i(x_0+x_5)]\}\end{aligned}$$

which represents a uniform matter distribution and a uniform electric field throughout space-time. Just as space curvature condenses light and neutrino radiation into quanta, so also it condenses such a static wave into wave packet particles. Put

$$\begin{aligned}&W' = \tfrac{1}{2}(1 + X_0 X_5)\, W, \qquad W'' = \tfrac{1}{2}(1 - X_0 X_5)\, W\\ &W = W' + W''\end{aligned} \tag{9.1}$$

First will be obtained the effect of a static gravitational field on the wave equation.

Operating on the equation $\operatorname{curl}_5 W = 0$ with operators $\tfrac{1}{2}(1 + X_0 X_5)$ and $\tfrac{1}{2}(1 - X_0 X_5)$, using

$$\nabla = X_1 \frac{\partial}{\partial x_1} + X_2 \frac{\partial}{\partial x_2} + X_3 \frac{\partial}{\partial x_3}$$

$$\begin{aligned}&\nabla W' + \left(X_0 \frac{\partial}{\partial x_0} + X_5 \frac{\partial}{\partial x_5}\right) W'' = 0\\ &\nabla W'' + \left(X_0 \frac{\partial}{\partial x_0} + X_5 \frac{\partial}{\partial x_5}\right) W' = 0\end{aligned} \tag{9.2}$$

Also

$$X_0 W' = -X_5 W',\ X_0 W'' = X_5 W'' \tag{9.3}$$

Assume factors $\exp(ix_5)$ and $\exp(iEt)$. Put $x_0 = Vt$ where V is the gravitational potential. E, the frequency, is usually associated with the energy.

Then

$$\nabla W' + iX_5\left(1 + \frac{E}{V}\right)W'' = 0$$

$$\nabla W'' + iX_5\left(1 - \frac{E}{V}\right)W' = 0$$

As a first approximation, since V is nearly equal to unity, the factor $1/V$ can be ignored, to give

$$\begin{aligned}\nabla W' + iX_5(E + V)\, W'' &= 0\\ \nabla W'' + iX_5(-E + V)\, W' &= 0\end{aligned} \qquad (9.4)$$

These are equivalent to Dirac's equations for the relativistic electron.

For electrons the electric potential is more significant. Allowing for an electromagnetic field dx_5 is replaced by $(dx_5 + \gamma_{5i}\, dx_i)$ where γ_{5i} is the electromagnetic potential. For a stationary electron this is dominated by the electric potential γ_{50}.

Then

$$\left(X_5 \frac{\partial}{\partial x_5} + X_0 \frac{\partial}{\partial x_0}\right)$$

is replaced by

$$X_5\left(\frac{\partial}{\partial x_5} + \gamma_{50} \frac{\partial}{\partial x_0}\right) + X_0 \frac{\partial}{\partial x_0}$$

Assuming

$$\frac{\partial}{\partial x_5} = i, \qquad \frac{\partial}{\partial x_0} = iE, \qquad 1 + \gamma_{50} E = V$$

this gives Dirac's equations as before

$$\begin{aligned}\nabla W' + iX_5(E + V)\, W'' &= 0\\ \nabla W'' + iX_5(-E + V)\, W' &= 0\end{aligned} \qquad (9.4)$$

We seek a wave packet solution of curl $W = 0$ which displays the properties usually associated with the electron. We start by assuming a potential of the form.

$$V = 1 - \frac{1}{r^2}, \qquad r^2 = (x_1)^2 + (x_2)^2 + (x_3)^2$$

with $E = 1$, equations (9.4) give approximately

$$\begin{aligned}\nabla W' + 2iX_5\, W'' &= 0\\ \nabla W'' + \frac{i}{r} X_5\, W' &= 0\end{aligned} \qquad (9.5)$$

and hence

$$\nabla^2 W' + \frac{2}{r} W' = 0 \tag{9.6}$$

Taking W' as a solution of this equation, with

$$W'' = \tfrac{1}{2} i X_5 \nabla W'$$

then $W = W' + W''$ is the required solution.

If R is any function of r,

$$\nabla(RW) = R \nabla W + R' \vartheta W, \tag{9.7}$$

where

$$\vartheta = x_i X_i / r \qquad (i = 1, 2, 3), \qquad \vartheta^2 = 1 \tag{9.8}$$

Take $V_1 = \Omega_1$ as any constant spinor, initially as

$$\tilde{\Omega} = [1, 0, 0, 0]$$

and

$$V_2 = \vartheta V_1$$

This gives

$$\vartheta V_1 = V_2, \qquad \vartheta V_2 = V_1, \qquad \nabla V_1 = 0, \qquad \nabla V_2 = \frac{2}{r} V_1 \tag{9.9}$$

Put

$$W_0 = R(V_1 + V_2) \tag{9.10}$$

where R is a solution of

$$R'' + \frac{2}{r} R' + \frac{2}{r} R = 0 \tag{9.11}$$

Then

$$\nabla W_0 = \frac{2R}{r} V_1 + R'(V_1 + V_2)$$

$$\nabla^2 W_0 = \left(\frac{2R'}{r} - \frac{2R}{r^2} \right) V_2 + \frac{2R'}{r} V_1 + R''(V_1 + V_2)$$

and

$$\left(\nabla^2 + \frac{2}{r} \right) W_0 = \frac{-2R}{r^2} V_2 \tag{9.12}$$

Since this is an order of magnitude smaller than $(2/r) W_0$, for large r_1 this gives a first approximation to the solution of (9.6). More exact solutions may be obtained by successive approximations.

Such a solution does not, however, give the potential $1 - 1/r$.

A second similar solution can be obtained in the same manner, taking the fixed spinor as

$$\tilde{\Omega}_2 = [0, 1, 0, 0]$$

which is associated with the factor $\exp(-ix_5)$ instead of $\exp(ix_5)$. This leads in the same way to W_2' and W_2'', remembering that W_2'' takes an extra minus sign since $\partial/\partial x_5 = -i$.

The combined solution is

$$W = (W_1' + W_1'') \exp[i(x_5 + t)] + (W_2' + W_2'') \exp[-i(x_5 + t)] \quad (9.13)$$

The terms of W'' are small compared with the terms of W', and as a first approximation can be ignored. The function $R(r)$ can be asymptotically either of the form

$$\exp(ik\sqrt{r})/r \qquad \text{or} \qquad \sin(k\sqrt{r} + k')/r$$

In the former case the factor $\exp(ik\sqrt{r})$ does not affect the concomitants. The factor $\frac{1}{2}(I + X_0 X_5)$ annihilates the magnetic terms in the 6-vector concomitants. The field is thus the sum of the electric components of the two spinor fields

$$\left[\frac{1}{r} + \frac{x_1}{r^2}, \frac{-x_2}{r^2}, 0, \frac{-x_3}{r^2}\right] \qquad \text{and} \qquad \left[\frac{-x_2}{r^2}, \frac{1}{r} - \frac{x_1}{r^2}, \frac{-x_3}{r^2}, 0\right] \quad (9.14)$$

which gives

$$E_1 = -4x_1/r^2, \qquad E_2 = -4x_2/r^2, \qquad E_3 = -4x_3/r^2 \quad (9.15)$$

which is the correct field for the classical electron. This gives the correct electric potential $1 - k/r$, and similarly for the gravitational potential.

This is the asymptotic solution. For the behaviour at the origin, suppose that near the centre of the electron

$$E^2 - V^2 = k^2$$

Then curl $W = 0$ gives

$$[\nabla^2 + k^2]\, W = 0$$

For Ω a constant, spinor solutions are given by

$$W_0 = \Omega \exp(ikr)/r \qquad \text{and} \qquad W_0 = \Omega \sin kr/r$$

The former has a point singularity, and should be excluded. The latter is regular at $r = 0$, and should be chosen for a wave packet solution.

This suggests, however, that in the asymptotic solution one should be chosen involving $\sin(k\sqrt{r} + k')$ instead of $\exp(ik\sqrt{r})/r$. This implies that the concomitants include an oscillating factor $\sin^2(k\sqrt{r} + k')$. There does not appear to be any experimental reason why this modification should not be correct.

The fields of the major portion W' have been studied. There remain the fields involving W'', the next significant terms involving $\tilde{\tilde{W}}$ terms of the form $\tilde{\tilde{W}}'\psi W''$.

If the asymptotic form involves $\exp(ik\sqrt{r})/r$ or $\exp(-ik\sqrt{r})/r$, there is an inward or outward flow of momentum. This would suggest that in a stable electron there is a balance between an outflow of energy and an inflow of energy. The oscillatory factor $\sin^2(k\sqrt{r}+k')$ corresponding to the interaction between these two. The combined term $\sin(k\sqrt{r}+k')$ leads to no radial momentum.

There are still momentum terms of the form

$$(2R/r^2+2RR'/r)[0, x_3, -x_2] \tag{9.16}$$

which would account for mechanical spin. Also there are magnetic fields

$$(2R^2/r+RR')[-1,0,0]$$

and

$$RR'/r[-x_1{}^2+x_2{}^2+x_3{}^2, -2x_1x_2, -2x_1x_3] \tag{9.17}$$

which would account for magnetic spin.

It would thus seem that there is a wave packet solution of $\operatorname{curl} W=0$ with properties essentially similar to the physical properties of the electron.

10. *Baryons*

It has been shown that the gravitational field of a photon acts as a wave guide to contain the light wave. If the gravitational field were sufficiently strong, therefore, it would be possible for this gravitational wave guide to be curved instead of straight. In particular, if the gravitational potential were sufficiently high the wave could follow a circular track.

The essential feature would be the induced gravitational field, and the light would not necessarily be monochromatic. The phase of the spinor components would change as the light was deflected round the track. One circumference would correspond to a rotation through 2π. The corresponding spinor phase would change by π, which would introduce a change of sign. To counterbalance this, the circumference of the circle would have to correspond to an odd number of half periods. Subject to the restriction that the wave lengths of all light should be an odd submultiple of twice the circumference of the circle, this light could have varying frequencies, which might account for the strong interaction of baryons.

Such a structure would have no charge, i.e., it could describe a neutron. However, if the circular gravitational wave guide were established, there is no reason why there should not be a wave involving $\exp(ix_5)$ propagated in one direction round the circle and a wave involving $\exp(-ix_5)$ in the reverse direction. Without going into details here, this would produce the electric field of a charged particle, and would make the complete wave packet follow the correct geodesic path for a charged particle.

Such a structure could describe a proton.

This suggested structure for a baryon, corresponds in many ways to Wheeler's Geons. Wheeler (1955), however, is concerned with macroscopic rather than microscopic phenomena. He dismisses possible application to the micro world on the grounds that it is 'far below the limit where it might be right to use classical theory'.

11. *Other Physical Correspondences*

Superposition and Resonance

Consider the superposition of two spinor fields

$$W = W_1 + W_2$$

The concomitants are not linearly additive, thus

$$\tilde{W}\psi W = \tilde{W}_1 \psi W_1 + \tilde{W}_2 \psi W_2 + \tilde{W}_1 \psi W_2 + \tilde{W}_2 \psi W_1$$

Thus if two electromagnetic fields $\tilde{W}_1 TX_i X_j W_1$ and $\tilde{W}_2 TX_i X_j W_2$ are superimposed, in the combined field, besides the linear sum of the two fields there appear the cross products $\tilde{W}_1 TX_i X_j W_2$ and $\tilde{W}_2 TX_i X_j W_1$.

If W_1 involves a factor $\exp(inx_0)$ and W_2 a factor $\exp(imx_0)$, then the cross products will involve factors $\exp[+i(n-m)x_0]$. Hence unless $n = m$ both cross products will oscillate about a zero mean. If these rapid oscillations are ignored, the electromagnetic fields become linearly additive.

If the exceptional case when $n = m$ the resulting field of two equal fields, for example, can vary between twice the sum and zero, according to the mutual phase. This is the effect of resonance.

Stationary Orbits

Suppose an electron describes an orbit about a nucleus with period of rotation an exact multiple of the period of oscillation of the electron. After one rotation the phase of the spinor returns to its original value, and a semi-stable equilibrium will be maintained.

If, however, the rotation period were not an exact multiple of the oscillation period then the phase of the spinor at this point would change for every revolution. This changing phase of the spinor would cause some radiation. Interference between the previous and the new phases would induce momentum and electromagnetic fields causing the orbit to become unstable.

Hence stable orbits can only be obtained when the rotation period is an exact multiple of the electron period. This model conforms with the general principles of quantum theory.

Energy and Frequency

The potential energy of an electron could be defined as the potential at such points for which

$$\left(X_0 \frac{\partial}{\partial x_0} + X_5 \frac{\partial}{\partial x_5} W = 0\right)$$

This makes the frequency of a stationary electron exactly equal to its potential energy. For a moving electron, since energy and frequency transform in the same way for a Lorentz transformation, the frequency must remain equal to the total energy, potential and kinetic.

Frequency of Spectral Lines

If an electron describes a stable orbit about a nucleus with energy E_1 and spinor wave function W_1, and then moves to another orbit with energy E_2 and wave function W_2, then W_1 involves a factor $\exp(iE_1 x_0)$, and W_2 a factor $\exp(iE_2 x_0)$.

For simplicity suppose that the wave during the transition is of the form

$$W = (1-k)\,W_1 + kW_2$$

where k increases from 0 to 1.

Concomitants of the form $\widetilde{\bar{W}}_1 \psi W_1$ and $\widetilde{\bar{W}}_2 \psi W_2$ correspond to stable orbits and do not involve radiation. But there are terms

$$(k-k^2)(\widetilde{\bar{W}}_1 \psi W_2 + \widetilde{\bar{W}}_2 \psi W_1)$$

which could cause radiation. The term $\widetilde{\bar{W}}_1 \psi W_2$ involves a factor $\exp[i(E_2 - E_1)x_0]$ while $\widetilde{\bar{W}}_2 \psi W_1$ involves a factor $\exp[-i(E_2 - E_1)X_0)]$.

Taken together these terms correspond to a frequency $E_1 - E_2$ which gives the correct frequency rule for spectral lines.

The Pauli Exclusion Principle

Two electromagnetic fields are approximately additive except when the frequencies of the sources of the fields are exactly in resonance. Concerning the interaction between the fields of two electrons this resonance can only occur if the two electrons have exactly the same energy. This is only likely to occur if the two electrons are confined to orbits about a nucleus corresponding to the same energy level so that this energy is precisely defined to be exactly equal for the two electrons.

If two electrons, then, occupy the same energy level orbit about a nucleus, then resonance will occur. Exceptional electromagnetic forces will result causing a strong interaction between the two electrons. There would be no doubt that at least one of the electrons would be ejected from the orbit. This would account for the Pauli exclusion principle.

12. *Conclusion*

A mathematically precise model may be constructed by considering a distribution of basic spinors in a Kaluza 5-space, such that

(1) The basic spinors W satisfy

$$\mathrm{curl}_5\, W = 0$$

(2) The four-dimensional curvature is in accordance with Einstein's General Theory, taking the energy–momentum–tension tensor as

$$T_{ij} = k_1 Q_i Q_j + k_2 P_i P_j + k_3 g_{ij}(P^k P_k)$$

where $Q_i = \tilde{\bar{W}}TX_i W, P_2 = \tilde{\bar{W}}T\phi X_i W$, and the ratios $k_1:k_2:k_3$ are adjustable to give the best fit.

(3) The five-dimensional curvature associated with the electromagnetic field is determined by the concomitant

$$F_{ij} = \tilde{\bar{W}}TX_i X_j W$$

In this model there can exist wave packets which behave in exactly the same way as the physical properties of photons, neutrinos, electrons and baryons.

This model is certainly a very close fit to the physical world as we know it.

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